(b) *Inelastic scattering* : In this case y is the same as x. But it has different energy and angular momentum, so that the residual nucleus Y (= X) is left in an excited state. The process can be written as X(x, y)X*, where the asterisk on X indicates an excited state of X.

(c) *Radiative capture* : In this case the projectile x is absorbed by the target nucleus X to form the excited compound nucleus (C*) which subsequently goes down to the ground state by the emission of one or more γ-ray quanta. We can write the process as $X(x, \gamma)Y^*$, (Y = C).

(d) *Disintegration process* : We can represent the process as X(x, y) Y where X, x, Y and y are all different either in Z or in A or in both. The first nuclear transmutation observed by Rutherford is an example of this process : $^{14}N(\alpha, p)^{17}O$

(e) *Many body reaction* : When the kinetic energy of the incident particle is high, two or more particles can come out of the compound nucleus. If y_1, y_2, y_3, etc. represent these different particles, we can write the reaction equation as $X(x, y_1, y_2, y_3.....)Y$. Examples are $^{16}O(p, 2p)\,^{15}N$; $^{16}O(p, pn)\,^{15}O$, $^{16}O(p, 3p)\,^{14}C$ etc. When the energy of x is very high, a very large number of reaction products usually result (3 to 20 for example). Such reactions are known as *spallation reactions*.

(f) *Photo-disintegration*: In this case the target nucleus is bombarded with very high energy γ-rays, so that it is raised to an excited state by the absorption of the latter. The compound nucleus C* = X*. The reaction can be written as X(γ, y) Y.

(g) *Nuclear fission* : When X is a heavy nucleus and y, Y have comparable masses, the reaction is known as nuclear fission. An example is $^{238}U\ (n, f)$. (see Ch. XIV).

(h) *Elementary particle reactions* : These involve either the production of elementary particles other than nucleons or nuclei as a result of the reaction or their use as projectiles or both of these. Examples are :

$$p + p \rightarrow p + n + \pi^+ ;$$

$$\pi^- + p \rightarrow \pi^0 + n ;$$

$$p + \pi^0 \rightarrow K^0 + \Lambda^0 \text{ etc.}$$

These reactions are usually produced at extremely high energies which may be several hundred MeV or more.

(i) *Heavy ion reactions* : In these reactions the target nucleus is bombarded by projectiles heavier than α-particles. Various types of products may be produced. The reactions usually take place at fairly high energies (several hundred MeV) of the projectile. Examples are :

$$^{10}B\,(^{16}O, {}^4He)\,^{22}Na,$$

$$^{14}N\,(^{14}N, {}^{15}N)\,^{13}N \text{ etc.}$$

10.3 Conservation laws in nuclear reactions

The occurrence of a nuclear reaction is usually governed by certain conservation laws.

(a) Conservation of mass number : The total number of neutrons and protons in the nuclei taking part in a nuclear reaction remains unchanged after the reaction. Thus in the reaction X(x, y)Y represented by Eq. (10.1-2), the sum of mass numbers of X and x must be equal to the sum of the mass numbers of Y and y:

$$A + a = A' + a' \qquad ...(10.3\text{-}1)$$

In the general case of reactions involving elementary particles the law can be expressed by requiring the total number of heavy particles (baryons) remains unchanged in a reaction (see Ch. XVIII).

(b) Conservation of atomic number : The total number of protons of the nuclei taking part in a nuclear reaction remains unchanged after the reaction. This means that the sum of atomic numbers of X and x is equal to the sum of atomic numbers of Y and y :

$$Z + z = Z' + z' \qquad ...(10.3\text{-}2)$$

In view of the conservation law (a) and (b) above it is easily seen that the mass number and the atomic number of the product nucleus in Rutherford's experiment (Eq. 10.1-1) should be $A' = A + a - a' = 14 + 4 - 1 = 17$ and $Z' = Z + z - z' = 7 + 2 - 1 = 8$, so that the product nucleus must be the isotope ^{17}O of oxygen.

Further, in view of (a) and (b) the neutron number N remains unchanged in the reaction.

(c) Conservation of energy ; Q value of a nuclear reaction : In order to apply the law of conservation of energy in the case of a nuclear reaction, it is necessary to take into account the mass-energy equivalance predicted by the special theory of relativity. Conservation of energy requires that the total energy, including the rest–mass energies of all the nuclei taking part in a reaction and their kinetic energies, must be equal to the sum of the rest–mass energies and the kinetic energies of the products.

Writing M_X, M_x M_Y and M_y as the rest–masses of the different atoms in Eq. (10.1-2), their rest mass energies are M_Xc^2, M_xc^2, M_Yc^2 and M_yc^2 respectively, Denoting the kinetic energy by E we then get $M_Xc^2 + M_xc^2 + E_X + E_x = M_Yc^2 + M_yc^2 + E_Y + E_y$.

During the nuclear reaction, the target nucleus is usually at rest, so that $E_X = 0$. The above equation then becomes

$$M_Xc^2 + M_xc^2 + E_x = M_Yc^2 + M_yc^2 + E_Y + E_y \qquad ...(10.3\text{-}3)$$

The above energy balance equation is often written without the factor c^2 in the mass–energy terms, which means that the masses are expressed in energy units.

It may be noted that though the nuclear masses are involved in a nuclear reaction, it is possible to write the energy–balance equation in terms of the atomic masses, since the electronic masses cancel out on the two sides of the equation and the electronic binding energies can be neglected.

It may be noted that at relatively lower energies, the kinetic energy is given by the non-relativistic expresions : $E = Mv^2/2$. When the energies of the particles involved in the reaction are very high, as in the case of many elementary particle reaction, the relativistic expression for the kinetic energy must be used : $E = \sqrt{p^2c^2 + M_0^2c^4} - M_0c^2$. Here M_0 is the rest mass of the particle and $p = M_0v/\sqrt{1-\beta^2}$ is its linear momentum (see Ch. XVIII).

(*d*) *Conservation of linear momentum* : If $\mathbf{p}_X$, $\mathbf{p}_x$, $\mathbf{p}_Y$ and $\mathbf{p}_y$ represent the momentum vectors of the different nuclei taking part in a reaction, then the law of conservation of linear momentum gives

$$\mathbf{p}_X + \mathbf{p}_x = \mathbf{p}_Y + \mathbf{p}_y \qquad ...(10.3\text{-}4)$$

Eq. (10.3-4) holds in an arbitary frame of reference. In the laboratory frame of reference (L-system) in which the target nucleus is at rest $\mathbf{p}_X = 0$ and the above equation becomes

$$\mathbf{p}_x = \mathbf{p}_Y + \mathbf{p}_y \qquad ...(10.3\text{-}5)$$

In the frame of reference in which the centre of mass of the two particles before collision is at rest (C-system), we have to write $\mathbf{p}_X + \mathbf{p}_x = 0$, which gives $\mathbf{p}_Y + \mathbf{p}_y = 0$ *i.e.*, the centre of mass of the product particles is also at rest in this system.

(*e*) *Conservation of angular momentum* : In a nuclear reaction of the type X + *x* → Y + *y*, the total angular momentum of the nuclei taking part in the reaction remains the same before and after the reaction.

Let I_X, I_x, I_Y, I_y denote the nuclear spins (total angular momentum) of the nuclei X, *x*, Y and *y* respectively. Let l_X represent the relative orbital angular momentum of X and *x* (*i.e.*, in the initial state). Similarly l_Y denotes the relative orbital angular momentum of Y and *y* (*i.e.*, in the final state). Then according to the law of conservation of angular momentum, we must have.

$$I_X + I_x + l_X = I_Y + I_y + l_Y$$

Application of the law of conservation of the angular momentum taking into account the well-known quantum mechanical properties of the former leads to certain selection rules.

(*f*) *Conservation of parity* : Since the nuclear reactions discussed in this chapter take place due to the strong interaction in which parity is conserved, the parity Π_i before the reaction must be equal to the parity Π_f after the reaction.

Denoting the intrinsic parities of the nuclei taking part in the reaction by Π_X, Π_x, Π_Y and Π_y we get for the initial and final states of the reaction

$$\Pi_i = \Pi_X \Pi_x (-1)^{l_X}$$

$$\Pi_f = \Pi_Y \Pi_y (-1)^{l_Y}$$

The conservation of parity requires that

$$\Pi_X \Pi_x (-1)^{l_i} = \Pi_Y \Pi_y (-1)^{l_f}$$

Except in the cases of elementary particle reactions, the intrinsic parity need not be taken into account. Hence we get

$$(-1)^{l_i} = (-1)^{l_f}$$

Parity conservation results in certain selection rules which limit the possible nuclear reactions that may occur starting from a given initial state *i*. For example, in the case of elastic scattering *l* can change only by an even integer.

(*g*) *Conservation of isotopic spin* : Denoting the isotopic spin vectors for the initial and final states by $\boldsymbol{T}_i$ and $\boldsymbol{T}_f$, we have from the law of conservation of isotopic spin applicable in the case of strong interaction

$$\boldsymbol{T}_i = \boldsymbol{T}_f$$

Since for the reaction X + *x* → Y + *y*, $\boldsymbol{T}_i = \boldsymbol{T}_X + \boldsymbol{T}_x$ and $\boldsymbol{T}_f = \boldsymbol{T}_Y + \boldsymbol{T}_y$, we have

$$\boldsymbol{T}_X + \boldsymbol{T}_x = \boldsymbol{T}_Y + \boldsymbol{T}_y$$

Isotopic spin is a characteristic of the nuclear level. Hence the above conservation law can be used to identify the levels of the nuclei produced in the reaction. In particular if $T_x = T_y = 0$ (as for the deuteron or the α-particle), we must have $T_X = T_Y$.

This rule must be obeyed in reactions of the type (*d*, α) (*d*, *d*) (α, *d*), (α, α) etc. The rule has been verified for the nuclei ^{6}Li, ^{10}B and ^{14}N for $T = 0$ in the ground states.