$$\therefore \qquad K \simeq \frac{p^2}{2m}$$

Therefore, **for small velocities, the relativistic kinetic energy expression reduces to classical expression.**

❍ MASSLESS PARTICLES (Photons)

A particle which has zero rest mass (m_0) is called a massless particle. In classical physics, the existence of massless particle is impossible. However, in relativistic mechanics, a particle with zero rest mass can exist.

The relativistic total energy E of a particle of rest mass m_0 in terms of its momentum p may be expressed as $E = \sqrt{(m_0^2c^4 + p^2c^2)}$

For massless particle, $m_0 = 0$

$$\therefore \qquad E = pc \qquad \text{or} \qquad p = \frac{E}{c}$$

Since p is also equal to $(mv =) Ev/c^2$, therefore, $v = c$, that is **the velocity of massless particle is same as that of light in free space.**

The energy of the particle, $E = pc = mc^2$,

where m is the mass equivalent to energy E, that is, $m = E/c^2$.

Thus we concluded that **every massless particle has energy pc and momentum E/c, and moves with the velocity of light.** The mass of massless particle is equal to E/c^2. It means that **a massless particle has mass so long as it is in motion. On being stopped they cease to exist-**they are either absorbed completely or are changed into heat at the surface. Thus, **the massless particle having energy and momentum, and can exist only when they move with the velocity of light.** The photons and neutrinos are the best examples of massless particle. In addition, **gravitons** are also massless particle with zero rest mass

$$(m_0 = 0)$$

❍ RELATIVISTIC DOPPLER'S EFFECT

The relativistic Doppler's effect is the phenomenon of apparent change infrequency (or wavelength) of light due to relative motion between the emitter (or source) and the received (or observer) when relativistic effects are taken into account. Relativistic Doppler's effect is due to time dilation effect of special theory of relativity. They decrease the total difference in observed frequencies and possesses the required Lorentz symmetry.

When the observer is in relative motion along the direction of propagation of light and is moving towards or away from the source the longitudinal relativistic Doppler's effect is observed. On the other hand, transverse Doppler's effect is observed when the observer is in a direction perpendicular to the direction of relative motion. Infact, the transverse Doppler effect is the normal redshift or blue shift predicated by special theory of relativity. Thus, relativistic transverse Doppler effect is purely a relativistic phenomenon.

There is no transverse Doppler effect in classical physics ($v << c$). To study Doppler effect, suppose.

Doppler Effect in Relativity : Suppose a light source of frequency ν is placed at the origin of S-frame. An observer in S'-frame moving away from the source at speed v measures a frequency of ν'. We have to find a relation between v' and v.

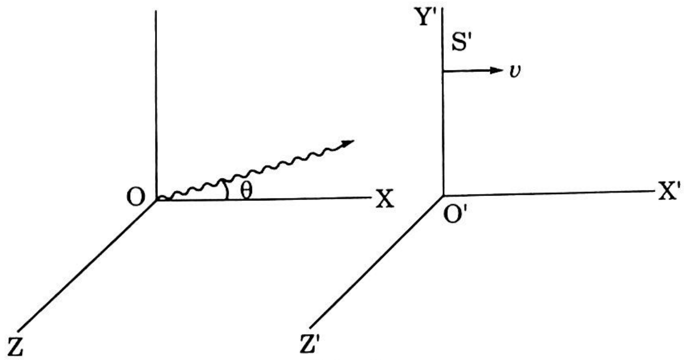

Fig. 8

A light wave were emitted from the source and moving in the $X - Y$ place making an angle θ with the x-axis would be represented by

$$\sin 2\pi\left(\frac{x\cos\theta + y\sin\theta}{\lambda} - \nu t\right), \qquad \text{...(i)}$$

where $\lambda\nu = c$, the velocity of wave propagation. The amplitude of the wave is supposed to be unity.

The expression describing the propagation in the S'-frame would be of the same form :

$$\sin 2\pi\left(\frac{x'\cos\theta' + y'\sin\theta'}{\lambda'} - \nu' t'\right), \qquad \text{...(ii)}$$

where λ' and ν' are the wavelength and frequency respectively as observed in S'. Again $\lambda'\nu' = c$, which is same for both the frames.

Let us apply Lorentz transformation equations to (i) by putting

$$x = \frac{x' + vt'}{\sqrt{(1-\beta^2)}}, \; y = y' \text{ and } t = \frac{t' + (x'v/c^2)}{\sqrt{(1-\beta^2)}},$$

where $\beta = v/c$. We obtain

$$\sin 2\pi\left(\frac{(x' + vt')}{\sqrt{(1-\beta^2)}}\frac{\cos\theta}{\lambda} + \frac{y'\sin\theta}{\lambda} - \nu\frac{t' + (x'v/c^2)}{\sqrt{(1-\beta^2)}}\right)$$

$$= \sin 2\pi\left(\frac{x'}{\sqrt{(1-\beta^2)}}\frac{\cos\theta}{\lambda} + \frac{vt'}{\sqrt{(1-\beta^2)}}\frac{\cos\theta}{\lambda} + \frac{y'\sin\theta}{\lambda} - \frac{\nu t'}{\sqrt{(1-\beta^2)}} - \frac{\nu x' v/c^2}{\sqrt{(1-\beta^2)}}\right)$$

$$= \sin 2\pi \left[\frac{x'}{\sqrt{(1-\beta^2)}} \frac{\cos\theta}{\lambda} + \frac{vt'}{\sqrt{(1-\beta^2)}} \frac{\cos\theta}{c/\nu} + \frac{y' \sin\theta}{\lambda} - \frac{\nu t'}{\sqrt{(1-\beta^2)}} - \frac{(c/\lambda)x' \, v/c^2}{\sqrt{(1-\beta^2)}} \right]$$

$$= \sin 2\pi \left[\frac{x'}{\sqrt{(1-\beta^2)}} \frac{\cos\theta}{\lambda} + \frac{\beta t' \nu \cos\theta}{\sqrt{(1-\beta^2)}} + \frac{y' \sin\theta}{\lambda} - \frac{\nu t'}{\sqrt{(1-\beta^2)}} - \frac{\beta x'}{\sqrt{(1-\beta^2)}} \frac{1}{\lambda} \right]$$

$$= \sin 2\pi \left[\frac{x'(\cos\theta - \beta)}{\lambda\sqrt{(1-\beta^2)}} + \frac{y' \sin\theta}{\lambda} - \frac{\nu t'(1-\beta\cos\theta)}{\sqrt{(1-\beta^2)}} \right]. \quad \text{...(iii)}$$

This must represent a propagation in S'-frame, and hence must be identical to eq. (i). Equating the coefficients of t' in (ii) and (iii), we get

$$\nu' = \frac{\nu(1-\beta\cos\theta)}{\sqrt{(1-\beta^2)}}, \quad \text{...(iv)}$$

This is the general expression for Doppler effect in relativity.

If $\theta = 0$, when the observer is in relative motion along the direction of propagation of light and is moving away from the source, we have

$$\nu' = \frac{\nu(1-\beta)}{\sqrt{(1-\beta^2)}} = \frac{\nu(1-\beta)}{\sqrt{[(1-\beta)(1+\beta)]}}$$

or

$$\nu' = \nu\sqrt{\left(\frac{1-\beta}{1+\beta}\right)} = \nu\sqrt{\left(\frac{c-v}{c+v}\right)},$$

which shows that the observed frequency ν' is lower than the actual frequency ν. If the observer is moving towards the source (v negative), then we would have

$$\nu' = \nu\sqrt{\left(\frac{c+v}{c-v}\right)},$$

which shows that the observed frequency ν' is higher than the actual frequency ν. This is the **"longitudinal" Doppler Effect** in light.

These results have been confirmed experimentally by **Ives and Stilwell** in 1938 who used a beam of excited hydrogen atoms of well-defined speed and direction as the source of light.

If the observer is in relative motion perpendicular to the direction of propagation of light, then also a Doppler effect is observed. Let us write eq. (iv) inversely as

$$\nu = \frac{\nu'(1+\beta\cos\theta')}{\sqrt{(1-\beta^2)}}.$$

Now putting $\theta' = 90°$, we get

$$\nu = \frac{\nu'}{\sqrt{(1-\beta^2)}}$$

or

$$\nu' = \nu\sqrt{(1-\beta^2)}.$$

Thus, if our line of sight is 90° to the relative motion, then the observed frequency ν' is lower than the actual frequency ν. This is known as **'Transverse Doppler Effect'** and has been confirmed experimentally. This is purely a relativistic phenomenon. There is no transverse Doppler effect in classical Physics ($\upsilon << c$). This can be checked by ignoring β^2 in the last expression which then gives $\nu' = \nu$.

❍ APPLICATIONS OF DOPPLER'S EFFECT

The Doppler's principle has, many application in astronomy. It is an important tool for deciding whether a particular star or galaxy is approaching towards the earth or receding from it. When light coming from a distant star or galaxy is observed by a high resolution spectrometer, the several well defined spectral lines are observed corresponding to the elements present in them. But compared with the corresponding spectral lines from a source on the earth, these lines show a slight frequency shift. By Doppler's effect we know that. when the star approaches the earth, the line is shifted towards the shorter wavelength, that is, towards the blue end of the spectrum. On the other hand, when the star or galaxy moves away from the earth, the lines are shifted towards the red end. By measuring the shift, the velocity of star can be calculated .It has been generally found that, the wavelength of light received from the star shifted slightly towards the red end. It shows that the stars are receding away from us and our universe is expanding.

The pair of stars which can not resolved even with a high power telescope appears as a single one, have been discovered by Doppler's principle. The pair of stars are called binaries. Some binaries consist of bright and dark stars and some consist of two bright stars each revolving about the common centre of their gravity. The first type of binary gives a single spectral line which shifts periodically about a fixed point while the second type gives double lines which coalesce periodically.

❍ GRAVITATIONAL REDSHIFT

Gravitational redshift or Einstein shift is the process by which electromagnetic radiation originating, from a source in a gravitational field is reduced in frequency (or increased in wavelength) or red shifted when observer is in a region of weaker gravitational field. Conversely, there also exists a corresponding blue-shift when electromagnetic radiation emerging from a light source in a weaker gravitational field propagates to a area of a stronger gravitational field.